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Wrinkling Analysis Using Improved Dynamic Relaxation Method

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The purpose of this paper is to propose a new method to reduce the calculation cost of the dynamic relaxation method. The proposed method combines the dynamic relaxation method with a static iterative method via two switching rules. In this method, the numerical scheme used in the analysis is selected as the situation of the analysis. An unstable region of the analysis in which the static iterative method cannot obtain the converged solution is performed by the robust dynamic relaxation method. Then a stable region that is costly to be solved by the dynamic relaxation method is performed by the efficient iterative method. By switching the analysis methods like this, the proposed method can use only the advantages of each method and complement each drawback. The performance of the new method is verified through a comparison of numerical analyses for the wrinkled membrane with the new method and conventional methods.

Nomenclature

C	=	damping matrix
D	=	diagonal matrix
E_{13}	=	component of antiplane shear tensor in 13 directions
E_{23}	=	component of antiplane shear tensor in 23 directions
e	=	base vector
F	=	external force vector
G_i	=	covariant base vector in the initial configuration
h	=	thickness of the element
K	=	tangent stiffness matrix
L	=	lower triangular matrix
M	=	mass matrix
Q	=	internal force vector
R_{tol}	=	tolerance in the relative residual force vector
r	=	natural coordinate
U_{tol}	=	tolerance in the relative displacement vector
u	=	displacement vector
V_3	=	director vector
α	=	coefficient for R_{tol}
α_c	=	critical damping constant
α^k	=	rotation freedom around V_1 at node k
β	=	coefficient for U_{tol}
β^k	=	rotation freedom around V_2 at node k

Subscript

i	=	variable number

Superscripts

A	=	indicating values in $(r_1, r_2) = (0, 1)$
B	=	indicating values in $(r_1, r_2) = (-1, 0)$
C	=	indicating values in $(r_1, r_2) = (0, -1)$
D	=	indicating values in $(r_1, r_2) = (1, 0)$
T	=	transpose
t	=	indicating values in the current configuration
0	=	indicating values in the initial configuration

I. Introduction

SOME space missions using the membrane (solar sail, space antenna, and so on) have been proposed because the membrane structure has some advantages as the space structure, such as ease of packaging and being lightweight. However, because the membrane has little compressive rigidity, when a compressive force is loaded, the membrane is easy to wrinkle. It is noted that a wrinkle changes the vibration characteristic of the membrane structure [1] or degrades surface accuracy. The vibration characteristic or surface accuracy is so important a design factor as to affect feasibility for some space missions. Thus, to predict the effects of wrinkles on the membrane, an accurate, efficient, and stable analysis method for the wrinkled membrane has been needed.

The analysis method for a wrinkled membrane is composed of a model to express wrinkles and a numerical scheme to obtain the solution in the nonlinear finite element analysis. Many models have been proposed as the model to express wrinkles. They are roughly classified into two groups: the tension field (TF) theory and the bifurcation buckling (BB) theory. In the TF theory, the membrane is assumed to have no compressive rigidity in the normal direction to wrinkles. Therefore, a stress in the normal direction to wrinkles is set to be zero and the stress field in wrinkled regions is in a uniaxial tension state. To realize the stress and strain field based on the TF theory, some methods that modified the constitutive relation have been proposed and implemented in the finite element analysis [2–5]. The TF theory uses membrane elements in the finite element analysis. Thus, the analysis gives details of in-plane deformation of wrinkles, wrinkled regions, and wrinkle directions; however, the analysis gives no information about out-of-plane deformation, amplitude, and wavelength of wrinkles.

In the BB theory, wrinkles are ascribed to bifurcation buckling phenomenon and analyzed by using the bifurcation buckling analysis in the finite element analysis. The analysis based on the BB theory uses shell elements. Thus, as the solution of the analysis, not only in-plane details of wrinkles, but also out-of-plane details, are obtained. However, generally speaking, simulation time for the analysis is longer than that in the TF theory because of the cost of the bifurcation analysis and higher degrees of freedom of shell elements

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than those of membrane elements. To reduce the calculation cost, the pseudo bifurcation buckling analysis, which does not need the bifurcation buckling analysis to obtain solutions on the bifurcation path, has been often used for the analysis of the wrinkled membrane [6–8].

On the other hand, some schemes have been proposed as the numerical scheme to obtain solutions. The dynamic relaxation (DR) method has been recently applied to investigate wrinkled membrane structures [9–11]. In the DR method, a static equilibrium solution is obtained as a converged solution of dynamic transient analysis. The DR method is more stable than other methods for wrinkling analysis to obtain the static equilibrium solution. However, the DR method has the drawback that a longer calculation time is required to obtain the converged solution. To overcome this drawback, some methods have been proposed. Most methods try to improve the damping effect in dynamic transient motion [12,13]. However, because the convergence speed of these methods depends on the damping effect in dynamic transient motion, drastic improvement of the convergence speed is difficult through these methods. Thus, another method to reduce the calculation time is needed.

The purpose of this paper is to propose a new numerical scheme for the wrinkled membrane that overcomes the drawback of the DR method. In this paper, a new DR method is proposed to let the solution converge rapidly without detracting from the high stability of the conventional DR method. The performance of the new method is verified through a comparison of numerical analyses for the wrinkled membrane with the new method and conventional methods.

II. Nonlinear Finite Element Analysis for a Wrinkled Membrane

To analyze the membrane, a finite element analysis including the geometrical nonlinearity is needed. In addition, the shell element is required to obtain the details of out-of-plane deformation of wrinkles. Thus, in this paper, the BB theory is used as the model to express wrinkles. To reduce the calculation time of the analysis based on the BB theory, the pseudo bifurcation buckling analysis is conducted. The details of the finite element analysis are described in the following subsections.

A. Formulation of Nonlinear Finite Element Analysis

In this paper, as the shell element, the mixed interpolation of tensorial components (MITC) shell element shown in Fig. 1 is used to avoid the shear locking [14,15]. In the MITC shell element, to avoid the shear locking, interpolation functions for components of the antiplane shear tensor are redefined:

$$E_{13} = \tfrac{1}{2}(1 + r_2)E_{13}^{A} + \tfrac{1}{2}(1 - r_2)E_{13}^{C} \quad (1)$$

$$E_{23} = \tfrac{1}{2}(1 + r_1)E_{23}^{D} + \tfrac{1}{2}(1 - r_1)E_{23}^{B} \quad (2)$$

By using the interpolation functions in Eqs. (1) and (2), even when a thickness of the membrane is very thin, the shear locking can be avoided [14,15]. In addition, to improve the convergence of the analysis including a finite rotation, the finite rotation tensor is introduced in the incremental calculation of the director vectors [16].

B. Pseudo Bifurcation Buckling Analysis

The pseudo bifurcation buckling analysis can obtain a solution on the bifurcation path without performing the bifurcation buckling analysis. Thus, to reduce the calculation cost, the pseudo bifurcation buckling analysis has been often used for the analysis of the wrinkled membrane [6–8]. In the pseudo bifurcation buckling analysis, an initial imperfection‡ is introduced instead of conducting the bifurcation buckling analysis. In this paper, as the initial imperfection, a random mode that has an amplitude of 10% or less of the thickness of an element is assumed. The initial imperfection is given as an initial out-of-plane displacement for each node. After the initial displacements are set like this, the normal analysis method should be conducted to obtain the solution.

‡Initial imperfection is a technical term.

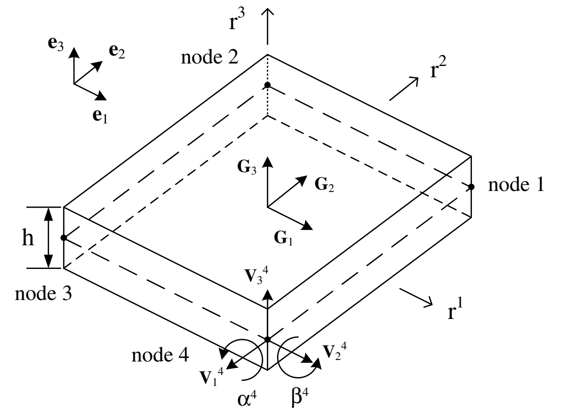

Fig. 1 Configuration of the MITC shell element.

III. Dynamic Relaxation Method

A. Basic Formulation

The DR method is applied to analyze various problems such as form-finding analysis and membrane-wrinkling analysis, because the DR method can stably solve problems including strong nonlinearity. The basic idea of the DR method is to assume that a solution of dynamic transient analysis converges to the static equilibrium solution. In general, the equation of motion of a structure is described next:

$$\boldsymbol{M}\,{}^{t}\ddot{\boldsymbol{u}} + \boldsymbol{C}\,{}^{t}\dot{\boldsymbol{u}} + {}^{t}\boldsymbol{Q} = {}^{t}\boldsymbol{F} \quad (3)$$

If a value of the damping matrix $\boldsymbol{C}$ is positive, the values of ${}^{t}\ddot{\boldsymbol{u}}$ and ${}^{t}\dot{\boldsymbol{u}}$ approach zero. As a result, the converged solution of dynamic transient analysis is obtained as follows:

$${}^{t}\boldsymbol{Q} = {}^{t}\boldsymbol{F} \quad (4)$$

Equation (4) is the same form as a static equilibrium solution. Thus, the static solution could be obtained as the converged solution of the dynamic transient analysis in the DR method. In the DR method, the value of ${}^{t}\ddot{\boldsymbol{u}}$ and ${}^{t}\dot{\boldsymbol{u}}$ would not be completely zero, although the value of them is nearly zero. The analysis should be stopped when the values of ${}^{t}\ddot{\boldsymbol{u}}$ and ${}^{t}\dot{\boldsymbol{u}}$ are enough small to obtain the accurate solution. In this paper, whether the analysis should be stopped or not is judged according to the following convergence criteria:

$$\frac{\|{}^{t}\boldsymbol{u} - {}^{t-\Delta t}\boldsymbol{u}\|}{\|{}^{t}\boldsymbol{u}\|} \le U_{\mathrm{tol}} \quad (5)$$

$$\frac{\|{}^{t}\boldsymbol{F} - {}^{t}\boldsymbol{Q}\|}{\|{}^{t}\boldsymbol{F}\|} \le R_{\mathrm{tol}} \quad (6)$$

Here, if the analysis has no external force vector, Eq. (6) cannot be calculated. In this case, the following criterion is used instead of the criterion in Eq. (6):

$$\frac{\|{}^{t}\boldsymbol{F} - {}^{t}\boldsymbol{Q}\|}{\|{}^{0}\boldsymbol{F} - {}^{0}\boldsymbol{Q}\|} \le R_{\mathrm{tol}} \quad (7)$$

The tolerances in Eqs. (5–7), U_{tol} and R_{tol}, are determined considering the balance between the calculation cost and the accuracy of the solution. However, in general, the DR method requires much longer calculation time to obtain a sufficiently accurate solution. This is a serious drawback in solving a large-scale system that has many degrees of freedom. Thus, a method to reduce the calculation cost of the DR method is needed.

B. Conventional Methods to Reduce the Calculation Time of the DR Method

To obtain rapid convergence to the static solution, some methods have been proposed. One of them is the adaptive damping method proposed by Zhang and Yu [12]. In the adaptive damping method, the damping matrix $\boldsymbol{C}$ is calculated as follows:

$$\boldsymbol{C} = \alpha_c \boldsymbol{M} \tag{8}$$

The critical damping constant to satisfy the critical damping varies with the change of displacement vector in the nonlinear analysis. Therefore, α_c is approximated at each step as follows:

$$\alpha_c \simeq 2\left(\frac{{}^t\boldsymbol{u}^T\,{}^t\boldsymbol{Q}}{{}^t\boldsymbol{u}^T\boldsymbol{M}\,{}^t\boldsymbol{u}}\right)^{\frac{1}{2}} \tag{9}$$

By setting the critical damping constant α_c like this, the adaptive damping method realizes the critical damping. On the other hand, Cundall [13] proposed another damping method: the kinetic damping method. This method does not use the damping effect of the damping matrix to obtain the converged solution. The procedure of this method is as follows:

1) The undamped motion of a system is calculated.

2) When the kinetic energy of the system approaches a peak, all velocity components are set to be zero.

3) The analysis is restarted at step 1 from the configuration at the peak step.

4) Until the static equilibrium solution is achieved, the process from step 1 to step 3 is repeated.

Through this process, the kinetic damping method can dissipate all modes of the transient vibration.

C. Time-Integration Method

Because the DR method needs to perform the time integration many times, a larger time step is desirable to reduce the number of the time integration and the calculation time. In the DR method, to set the larger time step in the time integration, mass matrix $\boldsymbol{M}$ is usually modified. In this paper, the modified mass matrix proposed by Hughes et al. [17] is used. Furthermore, the central-difference method is used for the time-integration method to avoid the calculation of the tangent stiffness matrix at each step.

IV. Improved Dynamic Relaxation Method

A. Consideration of Convergence of the DR method

As mentioned previously, though the DR method is robust, the calculation cost is high. Before a new method to reduce the calculation cost of the DR method is discussed, the reason why the calculation cost of the DR method is very high should be considered. In this section, Williams's toggle problem, as shown in Fig. 2, is discussed as an example. Input constants used in this problem are as shown in Table 1.

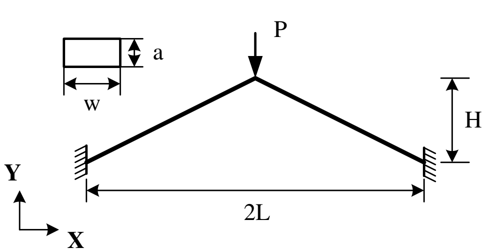

Fig. 2 Williams's toggle problem.

Table 1 Input constants

Input constant	Value
H	19.6 mm
L	328.57 mm
Toggle height a	6.17 mm
Toggle width w	19.13 mm
Young's modulus	72.32 GPa
Poisson's ratio	0.3
Density	1130 kg/m^3

In the model, a load P is applied to the center of the toggle structure. Both sides of the toggle structure are fixed. To analyze the problem, the DR method with the adaptive damping method was used. To consider the convergence of the DR method, the convergence history of the displacement in the Y direction at the center of the toggle structure is shown in Fig. 3, which also shows that about 98% deformation of the solution was obtained at 25% of the total step to obtain the solution. This result indicates that in the DR method, though the calculated solution is globally near the solution in the early stage of the analysis, it takes a long time to dissipate local fluctuations enough to obtain the accurate solution. This is why the DR method is costly. That is, it is found that to attenuate local fluctuations efficiently, reducing the calculation time of the DR method is required.

B. Improved DR Method

In this section, we propose to use the static iterative solution technique, especially the Newton–Raphson (NR) method to attenuate local fluctuations in the analysis. The NR method has rapid convergence speed when the configuration of the analysis is near the solution. Thus, the NR method is effective to attenuate local fluctuations. However, the NR method often cannot obtain the solution when the solution is far from the configuration of the analysis. In addition, the NR method easily becomes unstable when the analysis includes the bifurcation buckling, because the tangent stiffness matrix often becomes singular at the bifurcation buckling point. Therefore, when the NR method is used, the analysis must be near the solution and sufficiently stable.

In this paper, a new DR method that selects the numerical scheme, the DR method or the NR method, as the situation of the analysis is proposed. In the proposed method, an unstable region of the analysis in which the NR method cannot obtain the converged solution is calculated by the robust DR method. Then a stable region that is costly to be solved by the DR method is calculated by

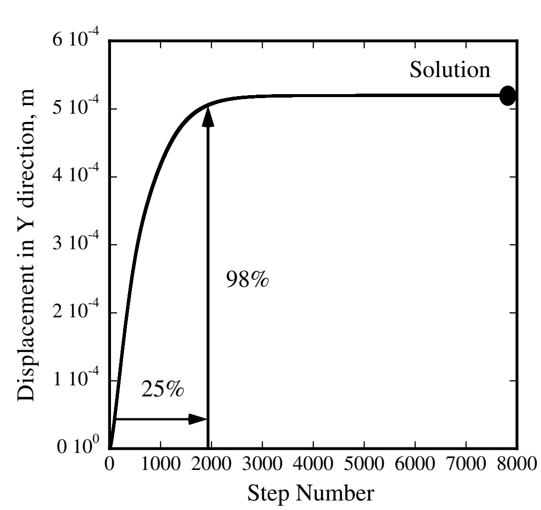

Fig. 3 Convergence history of the displacement in the *Y* direction at the center of the toggle structure.

the efficient NR method. The procedure of the proposed method is as shown in Fig. 4. The details of the procedure are described subsequently.

First, because the initial configuration of the analysis is assumed to be far from the solution, the DR method is performed until the analysis is roughly converged to satisfy Eqs. (5–7). When Eqs. (5–7) are satisfied, by assuming that the current configuration is in the stable region for the NR method, the analysis method is switched from the DR method to the NR method. Because it is not known whether the NR method can actually stably perform the analysis, the stability of the analysis with the NR method needs to be checked. Details of checking the stability of the analysis are described in the next section. When the analysis with the NR method is judged as unstable by checking the stability of the analysis, the analysis method is switched from the NR method to the DR method. This procedure is repeated until the converged solution is obtained in the NR method. Through the preceding procedure, the proposed method can use the high convergence speed of the NR method while avoiding the divergence of the analysis. Thus, the proposed method can reduce the calculation cost of the DR method without detracting the stability of the analysis. That is, the proposed method can use only the advantages of the DR method and NR method by complementing each drawback.

In addition, because the procedure of the DR method in the proposed method is the same as that in the conventional DR method, the proposed method can use the aforementioned conventional methods to improve the convergence speed of the DR method, such as the adaptive damping method and the kinetic damping method. In this paper, the kinetic damping method is used for the damping of the transient motion in the DR method.

C. Rule for Switching from the DR Method to the NR Method

In the proposed method, an important point is how to design two rules for switching: switching from the DR method to the NR method and switching from the NR method to the DR method. In this section, the rule for the switching from the DR method to the NR method is described. The switching is performed according to Eqs. (5–7). The details of the rules for switching are described subsequently.

The conventional DR method judges whether the analysis is stopped or not, based only on Eqs. (5–7). Thus, in the conventional DR method, it is necessary to set the tolerances in Eqs. (5–7) to be very small to obtain an accurate solution. However, in the proposed method, the DR method is used just to get a solution that is not far from the solution. Therefore, the tolerances do not need to be very small. Instead, large values are preferable for the proposed method. To reduce the total calculation time, it is preferable to switch from costly DR method to efficient NR method as soon as possible. Therefore, in this paper, the tolerances are set to be much larger values than those used for the conventional DR method. However, it is difficult to know the largest tolerances that ensure the stable analysis with the NR method. Thus, in this paper, the tolerances are determined adaptively as follows:

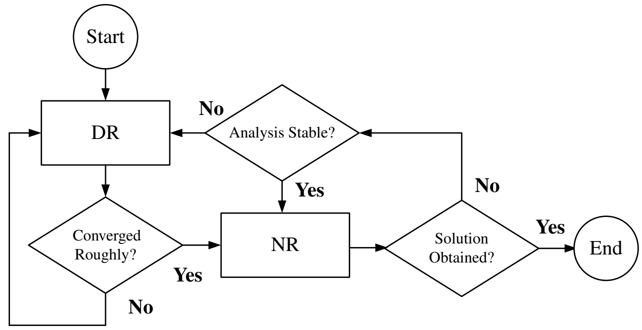

Fig. 4 Procedure of proposed method.

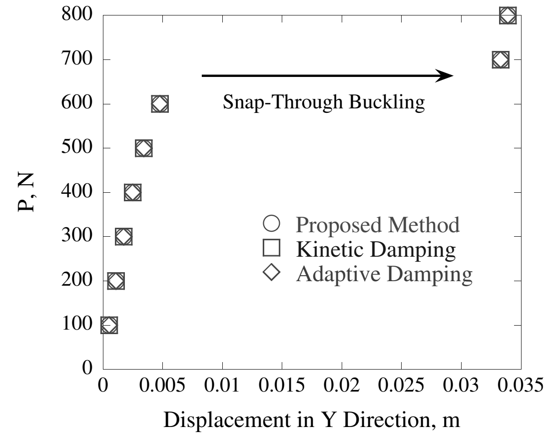

Fig. 5 Comparison of the displacement in the *Y* direction at the center of the structure.

$$R_{\mathrm{tol}}(\mathrm{new}) = \alpha \times R_{\mathrm{tol}}(\mathrm{old}) \qquad (\alpha < 1)$$
$$U_{\mathrm{tol}}(\mathrm{new}) = \beta \times U_{\mathrm{tol}}(\mathrm{old}) \qquad (\beta < 1) \qquad (10)$$

where $R_{\mathrm{tol}}(\mathrm{new})$ and $U_{\mathrm{tol}}(\mathrm{new})$ are tolerances to be set for the next DR analysis, and $R_{\mathrm{tol}}(\mathrm{old})$ and $U_{\mathrm{tol}}(\mathrm{old})$ are tolerances set for the previous DR analysis. Equation (10) shows that the tolerances are set to be smaller values than the previous setting values when the analysis with the NR method is judged to be unstable and the analysis is switched to the DR method again. Through this scheme, the values of tolerances can be adaptively set to be suitable values. The values of α and β in Eq. (10) should be set to be such small values that the analysis method is not switched many times. In this paper, the values of α and β are set to be 0.1.

D. Rule for Switching from NR Method to DR Method

There are two cases when the analysis method needs to be switched from the NR method to the DR method. It is well known that when the initial configuration[§] is very different from the solution, the NR method sometimes fails to obtain the solution, even if the iterative calculation is stably performed many times. This is the first case when the analysis method needs to be switched to the DR method. First, in this case, a rule for switching is considered. In general, when the NR method works normally, a rapid convergence for the solution can be expected. Therefore, even when the analysis with the NR method is not judged to be unstable, the analysis method is switched to the DR method when the iterative number of the iterative calculation exceeds a setting number.

The second case is when the analysis becomes unstable; in other words, it is when the tangent stiffness matrix becomes singular. The tangent stiffness matrix becomes singular when the analysis passes the buckling point. Because the analysis for the membrane includes many bifurcation buckling points in the narrow range, the analysis for the wrinkled membrane very easily becomes unstable. In this paper, the stability of the analysis is judged by checking a sign of a determinant of the tangent stiffness matrix. If the sign of the determinant is unchanged, it is assumed that the analysis is performed stably. On the other hand, if the sign of the determinant is changed in the process of the analysis, it is considered that the analysis is becoming unstable, and the analysis method is switched to the robust DR method. At this time, the determinant of the tangent stiffness matrix can be calculated from the diagonal matrix in the triangular factorization of the tangent stiffness matrix as follows:

$$\det(\boldsymbol{K}) = \det(\boldsymbol{L}\boldsymbol{D}\boldsymbol{L}^T) = \det(\boldsymbol{D}) = \prod_{i=1}^{n} D_{ii} \qquad (11)$$

[§]Initial configuration is a technical term.

Table 2 Comparison of the calculation time

P, N	Proposed method	Kinetic damping	Adaptive damping
100	1	2.8	7.3
200	1	4.0	12.1
300	1	4.5	12.4
400	1	7.0	18.8
500	1	11.6	39.3
600	1	5.4	29.3
700	1	2.7	7.8
800	1	30.7	76.1

The polarity of the determinant can be easily obtained by counting the number of negative elements in the diagonal matrix. Therefore, the increase of the calculation time for the judgment of the stability of the analysis at every step is negligible.

V. Numerical Simulations

The performance of the new method was verified through a comparison of numerical analyses for the wrinkled membrane with the new method and conventional methods. In the numerical simulations, to compare the calculation costs of each method fairly, the accuracy of the obtained results was matched in all methods. This was accomplished by setting the values of the tolerances of the NR method in the proposed method to the same value as those of the DR method in other methods. However, as the tolerance, the R_{tol} was only checked in the NR method, because the U_{tol} of the NR method in the proposed method did not have the same meaning as that in other methods. Of course, the DR method in the proposed method used both tolerances R_{tol} and U_{tol} in switching the analysis method from the DR method to the NR method. In the analysis, four-node MITC shell elements were used. In addition, the pseudo bifurcation buckling analysis was performed to reduce the calculation cost. In the DR method of the proposed method, the kinetic damping method was used for the damping of the transient motion in the DR method. Some numerical results are shown in the following sections. All numerical simulations were conducted on a Pentium D 3 GHz processor with 2 GB of RAM. The results of the following computation times were collected on this computer by measuring CPU processing times.

A. Williams's Toggle Problem

Williams's toggle problem was considered in Sec. IV.A. This problem was often used as the benchmark problem for the snap-through buckling. The details of the problem were the same as those described in Sec. IV.A. In the proposed method, the initial values of R_{tol} and U_{tol} were set to be 10^{-5}. The tolerances to obtain the solution were set to be 10^{-10} in the proposed method and the conventional methods.

First, the applied load P was changed from 100 to 800 N, and the results of the displacement in the Y direction at the center of the structure are shown in Fig. 5, which also shows that the proposed method obtained the same results as those obtained by the conventional methods. Moreover, in this problem, a snap-through buckling was observed between $P = 600$ and 700 N. However, the special modification for the NR method in the proposed method was not required to stabilize the analysis. Therefore, the proposed method could solve the problem stably, including the snap-through buckling. Next, the calculation time to obtain the solution in each method was compared. The results are shown in Table 2.

In Table 2, the values indicate the ratio of the calculation time against the calculation time in the proposed method. Table 2 shows that the proposed method can drastically reduce the calculation time compared with conventional methods, from one-third to one-thirtieth of the kinetic damping method and from one-seventh to one-eightieth of the adaptive damping method.

B. Simple Shear Deformation of the Rectangular Membrane

A simple shear deformation of the membrane shown in Fig. 6 was analyzed. In this problem, when a shear displacement δ was applied to the membrane, the whole area of the membrane was wrinkled. The upper side of the membrane was fixed except for the X direction, and the lower side was fully fixed. The sides of the membrane were free. The membrane was modeled with 3040 elements and 3168 nodes.

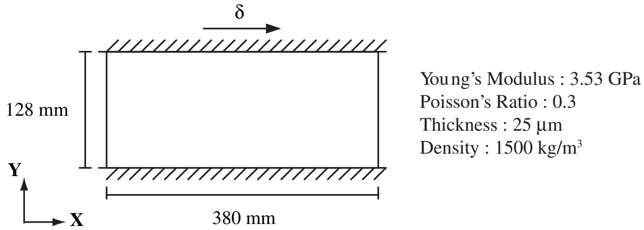

Fig. 6 Simple shear deformation of the rectangular membrane.

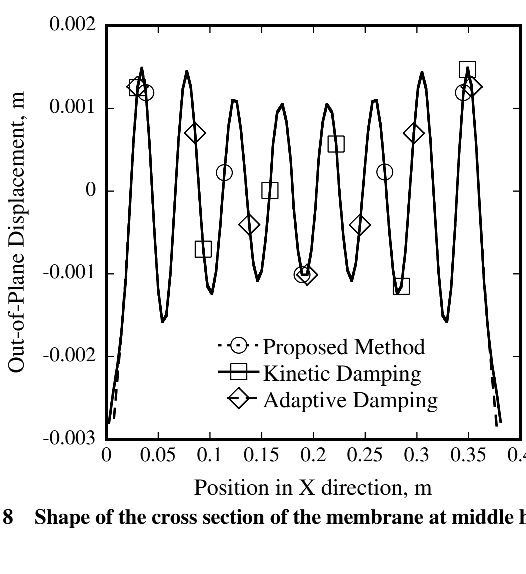

Fig. 8 Shape of the cross section of the membrane at middle height.

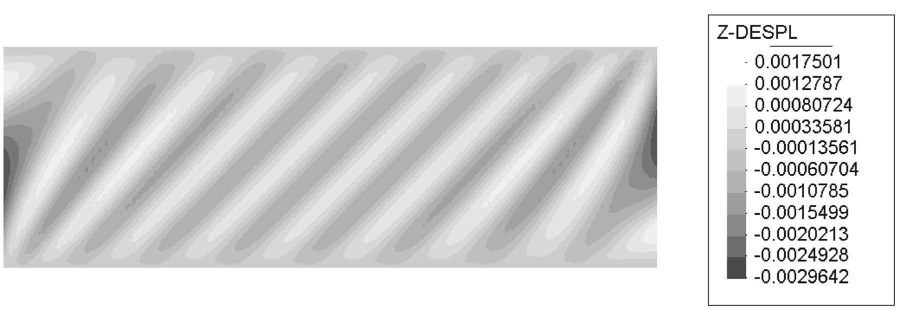

Fig. 7 Results of out-of-plane deformation of the membrane.

Table 3 Comparison of calculation time

	Proposed method	Kinetic damping	Adaptive damping
Calculation step	13,453 (11)	36,993	3,439,341
Calculation time, s	573.5 (44.6)	1499.1	237,442.1
Time ratio	1.0	2.6	414.0

[a]Value in parenthesis indicates costs of the NR method within total costs

The numerical simulations were conducted at $\delta = 3$ mm. In the proposed method, the initial values of R_{tol} and U_{tol} were set to be 10^{-7}. The final tolerances to obtain the solution were set to be 10^{-10} in the NR method of the proposed method and the conventional methods.

The result of the out-of-plane deformation obtained by the proposed method is shown in Fig. 7, in which the white region deforms to the front side and the black region deforms to the back side. In the figure, Z-DESPL shows the out-of-plane displacement. As the out-of-plane deformation, eight wrinkles were observed, counting peaks.

To compare the results of the proposed method with those of the other methods, the shape of the cross section of the membrane at middle height is shown in Fig. 8, which also shows no difference between the results by the different methods. This means that the results obtained by the proposed method had the almost same accuracy as those obtained by the conventional methods.

The calculation time and time steps to obtain the solution are shown in Table 3, which also clearly shows that the calculation time of the proposed method is drastically reduced compared with other methods, about one-third of the kinetic damping method and four-hundredth of the adaptive damping method.

C. Simple Tension Deformation of the Rectangular Membrane

A simple tension deformation of the rectangular membrane shown in Fig. 9 was analyzed. The left side of the membrane was fully fixed. The right side was moved by δ in the X direction and other degrees of freedom were fixed. The input constants used in the analysis were the same values as those in Sec. V.B. In the following numerical simulations, δ was set to be 25 mm. The membrane was modeled with 12,160 elements and 12,393 nodes. In the proposed method, the initial values of R_{tol} and U_{tol} were set to be 10^{-7}. The tolerances to obtain the solution were set to be 10^{-10} in the NR method of the proposed method and the conventional methods. Only the results by the proposed method and the kinetic damping method are described subsequently, because the adaptive damping method required a very long time to obtain the solution, and the analysis was forced to be stopped on the way.

The result of the out-of-plane deformation of the membrane by the proposed method is shown in Fig. 10, which also shows that the white region deforms to the front side and the black region deforms to the back side. As the out-of-plane deformation, wrinkles are observed in parallel with the X direction.

To compare the results of the proposed method with those of the kinetic damping method, the shape of the cross section of the membrane at $X = 190$ mm is shown in Fig. 11, which also shows no difference between the results by the different methods. This means that the results obtained by the proposed method had the almost same accuracy as those obtained by the kinetic damping method.

The calculation time and time steps to obtain the solution are shown in Table 4, which indicates that the calculation cost of the proposed method is less than one-fourth of the kinetic damping method and less than one-hundredth of the adaptive damping method.

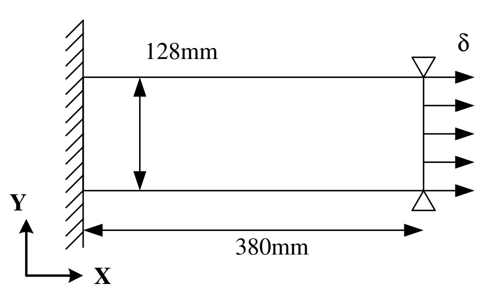

Fig. 9 Simple tension of the rectangular membrane.

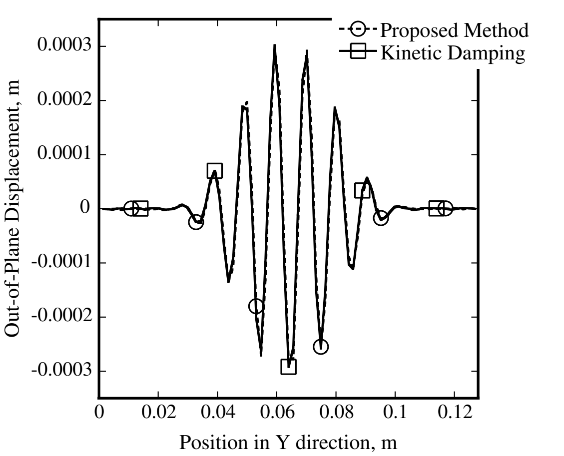

Fig. 11 Shape of the cross section of the membrane at $X = 190$ mm.

Fig. 10 Results of out-of-plane deformation of the membrane.

Table 4 Comparison of calculation time

	Proposed method	Kinetic damping	Adaptive damping
Calculation step	13,506 (8)	63,084	>1, 500, 000
Calculation time, s	4146.7 (244.8)	18,150.8	>500, 000
Time ratio	1.0	4.4	>100.0

[a]Value in parenthesis indicates costs of the NR method within total costs

VI. Conclusions

In this paper, a new method to reduce the calculation time of the dynamic relaxation (DR) method has been proposed. The proposed method combined the DR method with a static iterative method, and via two switching rules, the proposed method selected the numerical scheme as the situation of the analysis. By switching the numerical scheme as the situation of the analysis, the proposed method could use the advantages of each method and complement each drawback. The performance of the new method was verified through a comparison of numerical results for the wrinkled membrane with the new method and conventional methods. The comparison of numerical results indicated that the results obtained by the proposed method had the same accuracy as those of the conventional methods. In addition, the proposed method could reduce the calculation time drastically to obtain the same solution without detracting from the high stability of the DR method.

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